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Book Review

Applications of Lie Group Analysis in Geophysical Fluid Dynamics, by N. H. Ibragimov and R. N. Ibragimov, CNC Series on Complexity, Nonlinearity and Chaos, vol. 2, World Scientific, 2011; ISBN-13: 978-981-4340-46-5, ISBN-10: 981-4340-46-4

Andrzej Icha[1]

The book under review deals with the interplay between two subjects of applied mathematics, namely, nonlinear internal waves theory and the Lie group analysis of differential equations.

Internal gravity waves are the oscillations that occur in any fluid stably stratified with density, and belong to important objects of study in geophysical fluid dynamics (GFD). One general approach that is applicable to such nonlinear problems is the classical symmetries method originally proposed by Sophus Lie in 1881. From that time on, many excellent books have been dedicated to this subject and its generalizations. Given the authors' reputations, it is not unexpected that this book is also of high quality.

The book is divided into three parts, encompassing 10 chapters.

Part I, Internal Waves in Stratified Fluid, three chapters in total, starts with a very brief, qualitative introduction to the internal waves phenomenon. Chapter 2 contains basic notions and tools that enable the study of internal waves. Included are: basic GFD equations (without viscosity and diffusion), stratification, the linearization procedure and the equations of motion within the Boussinesq approximation. This chapter concludes with a detailed discussion of dispersion relation and the anisotropic nature of internal waves. The generation process of radiative internal waves by an oscillatory flow over a corrugated slope is discussed in chapter 3. Also, on the basis of the resonant triad model (RTM) developed by one of authors and explained in an appendix, Thorpe's problem is studied and analysed. (Thorpe's problem deals with the interaction of internal waves reflecting from slopes).

Part II, Introduction to Lie Group Analysis, comprises four chapters. Chapter 4 presents some elements of differential algebra (notations, definitions, differential functions, exact equations, etc.). These concepts are introduced in such a way that they are accessible to physicists.

The next chapter deals with transformation groups and some of their properties. Three illustrative examples of the method from elementary mathematics, classical mechanics and fluid mechanics are described, and the simplest transformations are explained (translations, isometry, dilatations, etc.). Subsequently, the classical one-parameter group theory is developed and culminated in Lie theorem, definitions of invariants and invariant equations, as well as in construction of groups in terms of canonical variables.

Chapter 6 concerns the symmetry analysis of differential equations. The basic idea of Lie's theory of symmetry (or admitted) group for differential equations concentrates on the invariance of the latter under a suitable transformation of independent and dependent variables. This mapping creates a local group of point transformations, establishing a diffeomorphism on the space of independent and dependent variables, transforming solutions of the equations to other solutions.

First, some useful notions and definitions are introduced. These include the generators of symmetry

[1] Pomeranian Academy in Słupsk, Institute of Mathematics, ul. Arciszewskiego 22d, 76–200 Słupsk, Poland. E-mail: majorana38@gmail.com

groups, the prolongation of group generators, and determining equations. Next, Lie symmetries of first- and second-order ordinary differential equations are discussed and calculated. The formal definition of Lie algebra with two examples is also presented. At the end of this chapter, the invariants of two-dimensional Lie algebra relating to the Euler equations in gas dynamics are calculated, together with the computation of the invariants of systems of first-order differential equations using canonical variables in low-dimensional Lie algebra.

In chapter 7, symmetry techniques applied to selected nonlinear differential equations are presented. The chapter begins with a discussion of the integration of two first-order differential equations using the canonical variables. Continuing on the same theme, the particular exact solutions of second-order ordinary differential equation are constructed. The next section is concerned with point symmetries of the Burgers equation and the system of shallow water equations, together with the methods for obtaining exact solutions of these equations. Finally, an excursion through the notion of conservation laws is performed. Classical Noether's theorem and new nonlocal conservation laws are discussed. This concept is a relatively recent and needs a further investigations.

Part III, about Group Analysis of Internal Waves, contains three chapters. It is concerned with the group analysis of nonlinear partial differential equations describing the motion of two-dimensional uniformly stratified rotating fluids. Such equations are used for investigating internal waves in uniformly stratified incompressible media.

Chapter 8 provides the mathematical background needed for the analysis of internal waves equations. Self-adjointness of basic equations is proved and used to find the conservation laws associated with symmetry of the system. The admitted Lie algebra for system, with and without rotation is also obtained. Chapter 9 continues this theoretical study and presents a concise discussion of conservation laws, both in the integral and differential forms. Some of the conservation laws related to symmetries of internal waves equations are trivial, but five conservation laws are significant.

The last chapter deals with the construction of the group invariant solutions to differential equations in question. These include exact solutions corresponding to the translation and dilatation groups of transformation, as well as the rotationally invariant solutions (for non-zero rotation of the Earth). The physical significance of these solutions remains open and needs an additional analysis.

An appendix, devoted to RTM in the theory of oceanic internal waves, provides a good supplement to the text. The set of coupled amplitude differential equations that governs the evolution of resonant triads on the slow interaction time scale is derived, and the numerical algorithm for the solution to these equations is described. Two model examples for resonant triad interactions are presented and discussed.

The book is rather well written and should be interesting to a broad spectrum of researchers, teachers and students: ocean physicists, atmosphere physicists and applied mathematicians. It presents some topics at a quite introductory level and treats other issues from an advanced point of view, so the basic courses in calculus and higher algebra, as well as in fluid mechanics, should provide an adequate prerequisite for the text. However, the reader should read this book with care, since there are several typographical errors and, linguistic misprints, especially in Part II. I am deeply convinced that the book will inspire some ambitious explorers to new ideas in geophysical fluid dynamics.